Journal of Engineering Physics and Thermophysics, Vol. 80, No. 3, 2007

FORMATION OF MAGNETIC FIELDS INSIDE MASSIVE CONDUCTING CYLINDERS

V. K. Goncharov, V. V. Makarov, and A. F. Chernyavskii

UDC 537.613

The penetration of strong magnetic fields into a cylindrical vacuum space with massive metal walls has been experimentally investigated. A practical solution on acceleration of the penetration of the magnetic field into the working space has been proposed.

Introduction. In experiments with high-current pulsed charged-particle beams, it frequently becomes necessary to form such beams in a vacuum space representing a metallic cylinder with fairly thick walls. If the pulse duration of the charged-particle flux is sufficiently long ($\sim 10^{-3}$ sec), it is necessary to form a quasistationary magnetic field with a duration of a few milliseconds inside the cylinder, i.e., to obtain magnetic induction in time in the form of a rectangular pulse. The simplest method of creating the magnetic field is to use an inductance coil around the cylinder. Formation of a 1–5 kA current in the form of a rectangular pulse in such a coil using an *LC* chain is quite a trivial problem. However, when there is a massive hollow semiconductor cylinder inside the coil, the coil–cylinder system changes to a transformer with a short-circuited turn and the process of mutual induction [1] keeps the magnetic field from penetrating rapidly into the cylinder.

Theoretical Substantiation. Let us consider a system consisting of two coils (one inside the other). The external coil usually has several tens of turns; the internal coil represents a brass cylinder fairly thick walls (~10 mm). It is necessary to evaluate the time of rise of the magnetic-field induction inside the system of coils to its maximum value upon switching on a direct current in the external coil. Since the evaluation in our case is only qualitative, we will use certain approximations for ideal systems, e.g., will employ formulas for coils of infinite length.

To obtain the values of magnetic inductions formed by each coil we consider fluxes which are created by the currents in the coils:

the flux created by the first coil and traversing the second coil is as follows:

$$\Psi_2 = Li_1 = N_2 B_1 S \,, \tag{1}$$

the flux created by the second coil and traversing the first one is

$$\Psi_1 = Li_2 = N_1 B_2 S \,. \tag{2}$$

From these formulas, we easily express the magnetic-field induction by the current strength. The sum of the magnetic-field inductions characterizes the change in the magnetic field inside this system; consequently, it is necessary to determine the manner in which the currents change in the first and second coils.

We write Ohm's law for the circuit of the first coil:

$$i_1 r_1 = U + E_{s1} + E_{m1} \,. \tag{3}$$

In this case we assume that the electromotive force (emf) of mutual induction E_{m1} may be disregarded (compared to the other terms); the emf of self-induction E_{s1} is determined as

---

A. N. Sevchenko Institute of Applied Physical Problems, Belarusian State University, 7 Kurchatov Str., Minsk, 220064, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 80, No. 3, pp. 177–180, May–June, 2007. Original article submitted November 2, 2006.

$$E_{s1} = -L_1 \frac{di_1}{dt},$$

and U is the supply voltage applied to the primary coil. As a result we obtain a nonhomogeneous differential equation of first order

$$\frac{di_1}{dt} + \frac{r_1}{L_1} i_1 = \frac{U}{L_1}, \tag{4}$$

the solution of which is the expression

$$i_1 = \frac{U}{r_1}\left(1 - \exp\left(\frac{t}{\tau_1}\right)\right), \tag{5}$$

where $\tau_1 = L_1/r_1$.

We write Ohm's law for the second coil in the form

$$i_2 r_2 = E_{s2} + E_{m2}, \tag{6}$$

here we have

$$E_{s2} = -L_2 \frac{di_2}{dt}, \quad E_{m2} = -L \frac{di_1}{dt} = -\frac{L}{L_1} U \exp\left(-\frac{t}{\tau_1}\right).$$

As a result we obtain a nonhomogeneous differential equation of first order

$$\frac{di_2}{dt} + \frac{r_2}{L_2} i_2 = \frac{E_{m2}}{L_2}. \tag{7}$$

Its general, the solution is the sum of the general solution $i_2 = D \exp(-t/\tau_2)$ of the corresponding homogeneous equation and the particular solution $i_2 = A \exp(-t/\tau_1)$ of the nonhomogeneous equation. With account for the initial conditions it takes the form

$$i_2 = \frac{LU}{L_2 r_1 - L_1 r_2}\left(\exp\left(-\frac{t}{\tau_1}\right) - \exp\left(-\frac{t}{\tau_2}\right)\right). \tag{8}$$

Now we may evaluate the magnetic induction inside the system of coils:

$$B = B_1 + B_2 = \frac{L}{S}\left(\frac{i_1}{N_2} + \frac{i_2}{N_1}\right). \tag{9}$$

Substitution of the expressions for currents determined by mathematical transformations and with allowance for the fact that $N_2 = 1$ yields a formula for analysis of the change in the magnetic induction inside the cylinder:

$$B = \frac{LU}{Sr_1}\left[1 - (1 - F)\exp\left(-\frac{t}{\tau_1}\right) - F \exp\left(-\frac{t}{\tau_2}\right)\right], \tag{10}$$

where

$$F = \frac{L}{N_1\left(L_2 - L_1 \frac{r_2}{r_1}\right)}.$$

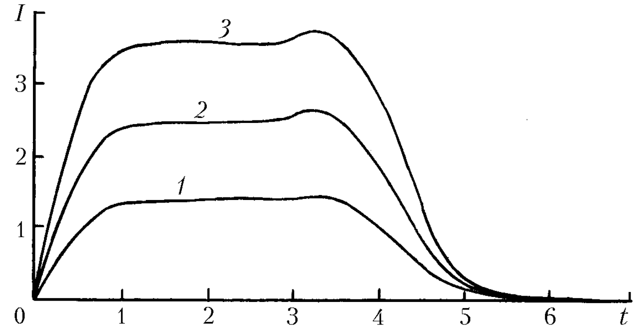

Fig. 1. Oscillograms of coil-current pulses for different voltages on the capacitors of the supply line: 1) U = 1, 2) 2, and 3) 3 kV. I, kA; t, msec.

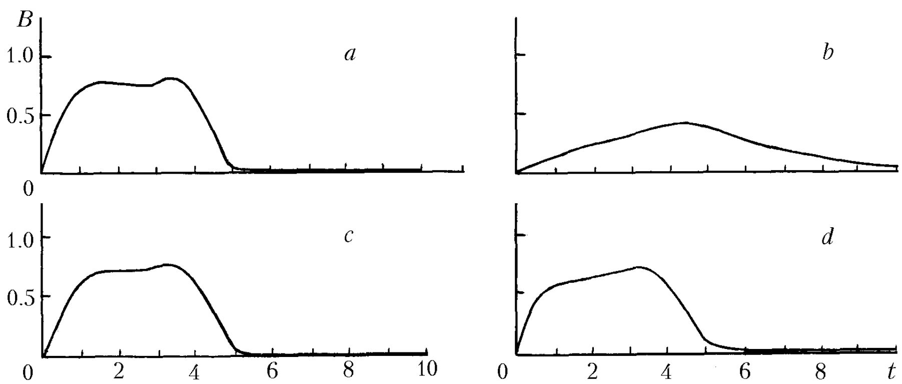

Fig. 2. Oscillograms of the magnetic induction at the center of the coil under different conditions: a) without a brass cylinder; b) with a massive brass cylinder; c) with a 0.35-mm-thick brass cylinder; d) with a cut massive brass cylinder. B, T; t, msec.

In our case the resistance of the thick-walled brass cylinder r_2 is several orders of magnitude lower that the resistance of the primary coil r_1; therefore, F is close to unity, and $(1 - F)$ tends to zero. Then (10) will take the form

$$B \approx \frac{LU}{Sr_1}\left(1 - \exp\left(-\frac{t}{\tau_2}\right)\right). \tag{11}$$

From relation (11), it follows that the times of penetration of the magnetic field into the coil–cylinder system and of reaching the magnetic-induction maximum are determined by the quantity τ_2 and consequently by the parameters of the second coil r_2 and L_2. For the experiments, we used an L59-brass cylinder of length 250 mm (equal to the length of the primary coil), outside diameter 94 mm, and wall thickness 12 mm. Evaluations of τ_2 in this case yield a value of ~10 msec, whereas the characteristic time is $\tau_1 \approx 1$ msec.

It seems impossible to theoretically calculate the time of penetration of the magnetic field into the cylinder, even though it has no complex interior, since the conductivity of the cylinder is strongly dependent on the technology of manufacture of its material and these data are not available in the reference books as a rule; therefore, experimental measurements must be carried out in each particular case.

Experimental. The magnetic field inside the cylinder was created using an inductance coil (110 μH). The coil was wound on a Caprolon form, into which a hollow brass cylinder was tightly fitted. A line of LC chains was created for power supply of the coil; the line made it possible to produce a nearly rectangular current pulse of more than 5 kA at voltages to 5 kV. The current through the coil was monitored using a low-inductance shunt of resistance 0.035 Ω [2, 3]. Figure 1 gives the oscillograms of the coil-current pulses for different voltages on the capacitors of the supply line. The magnetic induction inside the coil was measured using a Hall probe [2, 3]. The oscillogram of

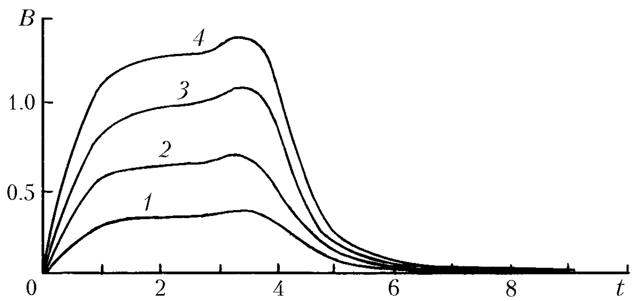

Fig. 3. Oscillograms of the magnetic induction at the center of the cut massive brass cylinder for different voltages on the capacitors of the supply line: 1) U = 1, 2) 2, 3) 3, and 4) 4 kV. B, T; t, msec.

magnetic induction at the center of the coil without a brass cylinder is shown in Fig. 2a. As is seen, the change in the magnetic induction inside the coil corresponds to the change in the coil current. However, when the brass cylinder is set into the coil, the magnetic induction changes slowly and does not reach its maximum even by the end of the current pulse (Fig. 2b). This is due to the fact that the rise in the magnetic induction of the basic coil is hindered by the magnetic field generated by the current of a short-circuited turn, i.e., of the brass cylinder. In such a case, for the quasistationary magnetic field with a duration of a few milliseconds to be attained, a very large capacitance is needed for the *LC* chain supplying the magnetic system; this leads to unacceptable weight-size parameters of the entire system. The higher the resistance of the brass-cylinder material, the more rapid the rise in the magnetic induction inside the cylinder. Figure 2c shows the oscillogram of the magnetic induction inside a 0.35-mm-thick brass cylinder. Here the pulse edges are not stretched strongly compared to the coil-current pulse (Fig. 2a). However, it is virtually impossible to obtain a mechanically strong vacuum system for charged-particle fluxes for such thicknesses, much less in the case of arrangement of certain beam-control systems inside the cylinder.

For the brass cylinder not to represent a short-circuited turn, we cut it lengthwise. The cut depth was 1 mm. The cut was filled with epoxy resin to produce vacuum in such a cylinder. The oscillogram of the magnetic-field induction inside the cut massive brass cylinder is given in Fig. 2d. As is seen in the figure, we are able to obtain a quasistationary magnetic field during ~3 msec. Measurements of the magnetic induction along the axis and radius of the cylinder have shown that the magnetic field is homogeneous throughout the cylinder. The longitudinal cut in the brass cylinder eliminates only transverse currents, which enables the magnetic field to rapidly penetrate into the cylinder; the longitudinal currents of removal of charged particles appearing on the cylinder wall are not hindered by it.

With the aim of checking the possibilities of controlling charged-particle beams, we measured the magnetic induction inside the cut massive brass cylinder for different voltages on the capacitors of the supply line (Fig. 3). As is seen in the figure, a quasistationary magnetic field with a magnetic induction of more than 1 T and a duration of ~3 msec was obtained.

Conclusions. The time of penetration of the magnetic field into a hollow conducting cylinder with fairly thick walls has theoretically been evaluated. It has been shown that the deceleration of the penetration of the magnetic field into the cylinder is determined by its conductivity and inductance where it carries transverse currents. A practical solution on acceleration of the penetration of the magnetic field into a thick-walled cylinder manufactured from a conducting material has been proposed. The longitudinal cut of the cylinder makes it possible to obtain, inside the coil–cylinder system, a quasistationary magnetic field of duration ~3 msec and magnetic inductance more than 1 T for minimum weight-size parameters. Such magnetic systems may be used for control of a relativistic-electron beam by a current with a pulse to several kiloamperes, which is necessary in creating free-electron lasers.

NOTATION

A and D, constants; B, magnetic induction, T; C, capacitance, F; E, emf, V; I, current measured experimentally in the circuit without the second coil, kA; i, current strength, A; L, mutual inductance, H; L_1, inductance of the first coil (primary inductance), H; L_2, inductance of the second coil (secondary inductance), H; N, number of turns; r, resistance, Ω; S, cross-sectional area of the coils, m^2; t, time, msec; U, supply voltage, V; τ, time constant of the discharge, sec; Ψ, magnetic-induction flux, Wb. Subscripts: 1, electric circuit without the second coil; 2, second coil; m, mutual induction; s, self-induction.

REFERENCES

1. I. V. Savel'ev, *A Course in Physics*, in 3 vols.: *Electricity. Oscillations and Waves. Wave Optics* [in Russian], Vol. 2, Nauka, Moscow (1982).
2. H. Knoepfel, *Pulsed High Magnetic Fields* [Russian translation], Mir, Moscow (1972).
3. W. Lochte-Holtgreven, *Plasma Diagnostics* [Russian translation], Mir, Moscow (1971).